NBU-003-016409 Seat No. ______

M. Sc. (Sem. IV) (CBCS) Examination

April / May - 2017

Mathematics : CMT-4001

(Linear Algebra)

Faculty Code : 003
Subject Code : 016409

Time : $2\frac{1}{2}$ Hours] [Total Marks : **70**

Instructions : (1) Answer all the questions.
(2) Each question carries 14 marks.

1 Answer any **seven** : **7×2=14**

(i) Define similar linear transformations and similar matrices for a finite dimensional vector space V over F.

(ii) Define an algebra over a field F. Also give an example of an algebra over the field F.

(iii) Let $T \in A_F(V)$ and $p(x) \in F[x] - \{0\}$. When $p(x)$ becomes a minimal polynomial of T over F ?

(iv) Let $T_1 S \in A_F(V)$. In standard notation verify that $r(ST) \le r(T)$.

(v) Define homomorphism between two algebras A_1, A_2 over a field F.

(vi) Let $A, B \in F_n$. In standard notation verify that $tr(A+B) = tr(A) + tr(B)$, where $tr : F_n \to F$ and it is trace of the matrix.

(vii) Let $T \in A_F(V)$. When T is said to a nilpotent element of $A_F(V)$?

(viii) Let $T \in A_F(V)$ and T is a nilpotent linear transformation. Define index on nilpotence for T.

(ix) Let $A \in \not\subset n$. When A is called Hermitian ? When A is called skew Hermitian ?

(x) Let $A \in F_n$ and A is invertible. Prove that $\det\left(A^{-1}\right) = \frac{1}{\det A}$.

2 Answer any **two** : **2×7=14**

(a) Let $T \in A_F(V)$ and T is invertible. Prove that T^{-1} has polynomial expression in T over F. i.e. there is $f(x) \in F[x]$ such that $T^{-1} = f(T)$.

(b) Let λ be a characteristic root of T, where $T \in A_F(V)$. Prove that $p(\lambda) = 0$, where $p(x) \in F[x]$ and it is a minimal polynomial of T over F.

(c) Let $T_1, T_2 \in A_F(V)$ and they both have same invariants. Prove that they are Similar.

(d) Let $A \in F_n$. Prove that the interchanging two rows of A change the sign of its determinant.

3 Answer any **one** : **1×14=14**

(a) Let $tr\left(T^k\right) = 0, \ \forall k \in \mathbb{N}$. Prove that T is nilpotent.

(b) Let $A, B \in F_n$. Prove that $\det(AB) = \det(A) \cdot \det(B)$.

(c) Let $\dim_F V = n$ and $L(V, V, F)$ is the vector space consisting, all the bilinear forms on V. Prove that $\dim_F\left(L(V,V,F)\right) = n^2$.

4 Answer any two : **2×7=14**

(a) State and prove - Cayley-Hamilton theorem.

(b) State and prove - Jacobson Lemma.

(c) Let V be a finite dimensional vector space over F and $T \in A_F(V)$. Prove that T is invertible iff the constant term of any minimal polynomial of T over F is non-zero term.

(d) Prove that the determinant of triangular matrix is equal to the product of its all the entries of the main diagonal.

5 Answer any two : **2×7=14**

(a) Let V be a finite dimensional vector space over F. Let $T \in A_V(F)$ and $p(x) \in F[x]$ be a minimal polynomial of T over F. Let $f(x) \in F[x] - \{0\}$ be such that $f(T)$ is a zero mapping. Prove that $p(x) / f(x)$ in $F[x]$.

(b) Let V be a finite dimensional vector space over F. Let $T \in A_V(F)$. Prove that there is a unique monic polynomial $g(x) \in F[x]$ of least degree such that $g(T)$ is a zero mapping.

(c) Let F be a field with char $F = 0$. Let $A = (\alpha_{ij})$, $B = (B_{ij}) \in F_n$. Let $\alpha_{ij} = 1, \forall i, j \in \{1, 2, ..., n\}$ and $\beta_{11} = n, \beta_{ij} = 0$, $\forall i, j \in \{1, 2, ..., n\}$ and $i \neq 1$ or $j \neq 1$. Verify that A and B are similar.

(d) Solve by Cramer's rule :

$$x_1 + 2x_2 + 3x_3 = -5,$$

$$2x_1 + x_2 + x_3 = -7 \text{ and}$$

$$x_1 + x_2 + x_3 = 0.$$